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**Working Paper 2014/27**

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# **What direction should the fishing mortality target change when natural mortality increases within an assessment?**

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## **ABSTRACT**

Traditionally, the natural mortality rate ( $M$ ) in a stock assessment has been assumed to be constant over years and ages. When  $M$  increases within an assessment, as has occurred in a number of Canadian cod stocks, the US Gulf of Maine cod stock, and the US Atlantic herring stock, the question arises how to change the fishing mortality rate target ( $F_{\text{target}}$ ). Yield per recruit considerations lead to an increase in the  $F_{\text{target}}$ , while maximum sustainable yield considerations often lead to a decrease in the  $F_{\text{target}}$ . Neither approach is theoretically superior. Using results from the recent Gulf of Maine cod assessment and an example from the Georges Bank yellowtail flounder assessment, both approaches are examined. Problems are found with both the yield per recruit and maximum sustainable yield approaches, leading us to recommend either not allowing  $M$  to change within an assessment model or if  $M$  does change to base the  $F_{\text{target}}$  on the natural mortality rate considered most appropriate based on the life history traits of the species of interest.

## **RÉSUMÉ**

## Introduction

The natural mortality rate ( $M$ ) in a stock assessment is a difficult parameter to estimate because it is not observed and is often confounded with other parameters within the model. Due to these difficulties in estimation, it has traditionally been assumed to be constant over years and ages. This has always been recognized as a simplifying assumption to allow other parameters to be estimated more precisely. The impacts of this assumption are small when  $M$  is small relative to the total mortality rate ( $Z$ ), but become more important as  $M/Z$  increases. This approach has evolved over time with the advancement of stock assessment models to allow for either estimation of a changing  $M$  over time and age, or else assuming a change in  $M$  based on hypothesized changes in predation or disease. For example, a number of Canadian cod stock assessments allow  $M$  to increase over time based on the lack of stock recovery under low fishing pressure (Chaput, 2011) while the Atlantic herring stock in US waters assumed a 50% increase in  $M$  to account for large increases in estimated consumption by a dozen finfish species (NEFSC, 2012a). Additionally, changes in  $M$  have been used to address retrospective patterns (Legault, 2009), for example in the Eastern Georges Bank cod assessment (Wang and O'Brien, 2012) and in the 2012 Georges Bank yellowtail flounder assessment (Legault et al., 2012). The US Gulf of Maine cod assessment uses both a constant  $M$  and  $M$ -change model to account for uncertainty in the underlying natural mortality (NEFSC, 2013). In the case of the Georges Bank yellowtail flounder assessment, the  $M$  change was abrupt from one value in the early years to a much higher value in recent years.

No matter the source of the change in  $M$ , the implication of the change on the target fishing mortality rate ( $F_{\text{target}}$ ) must be considered. There are two opposing possibilities when  $M$  increases within an assessment. One is to increase the target fishing mortality based on yield per recruit considerations (i.e., catch them before they die). Alternatively, the  $F_{\text{target}}$  is decreased based on life history and maximum sustainable yield considerations (i.e., the species can only withstand so much total mortality so an increase in  $M$  must be offset by a decrease in  $F$ ). Both of these approaches have a long history and theoretical basis, but have obviously different impacts on catch advice in the short term. The two approaches can best be compared and contrasted using examples.

### Gulf of Maine Cod Example

Stock assessments for the Gulf of Maine cod stock were conducted in 2011 (NEFSC, 2012b) and again in 2012 (NEFSC, 2013). The results of the two assessments were similar when  $M$  was assumed to be 0.2 for all years and ages. However, in the 2012 assessment, there was an additional model accepted which assumed  $M$  increased in a ramp pattern. The early years of the assessment (1982-1988) assumed  $M$  equaled 0.2, then increased linearly to 0.4 between 1989 and 2003, and remained at 0.4 for the remaining years (2004-2011). This model, denoted the  $M_{\text{ramp}}$  model, will be used as the basis for comparing the two approaches for estimating the target fishing mortality rate when the natural mortality rate changes within an assessment. The merits of the two accepted models are not considered in this paper.

The biological and fishery age vectors from the recent three years (2009-2011) indicate maturity occurs before entry to the fishery and that there are many ages accumulated in the plus group (Table 1). The plot of stock and recruitment estimates from this model does not indicate a strong relationship (Fig. 1). Thus, a proxy for  $F_{msy}$  was selected as  $F_{40\%}$ . For simplicity, in this paper the assessment stochastic projections to estimate MSY and  $B_{msy}$  are replaced by simply multiplying the yield and spawning stock biomass per recruit by the average recruitment from 1982-2009 (10.214 million fish).

The calculations of the MSY proxy reference points depend on the choice of  $M$ . The SARC Panel recommended using  $M$  of 0.2 under the assumption that the recent increase in  $M$  was only a temporary condition (NEFSC, 2013). However, the calculations can also be made assuming that  $M$  will remain at 0.4. The  $F_{msy}$  proxy of  $F_{40\%}$  and proxies for MSY and  $B_{msy}$  vary considerably depending on the value of  $M$  assumed (Table 2). As is typical, higher natural mortality rates result in higher  $F_{40\%}$  values. However, higher  $F$  and  $M$  result in lower equilibrium population abundance at age, as expected, with large changes in the contributions of older ages to spawning stock biomass (Table 3). Note that all eight cases assume the same recruitment at age 1, meaning that expected recruitment is independent of spawning stock size even at the lowest levels observed in the table. At a given  $F$ , the spawning stock biomass per recruit is always lower for larger  $M$ , while the spawning potential ratio is always higher for larger  $M$  (Fig. 2).

The replacement lines for these four  $M$  values plotted on the stock recruitment estimates show graphically the expected changes in SSB at equilibrium (Fig. 3). This plot also demonstrates a problem with the stock recruitment relationship when  $M$  is changing. Specifically, if  $M$  increases rapidly at the end of the time series, the recruitment estimates can end up falling well below the unexploited replacement line (not shown). This is because the replacement lines assume entire cohorts have been subjected to the conditions used to create the SPR values. Since no cohorts ever experience constant conditions over their lifespan, except in simulations, replacement lines can be misleading and should be interpreted with caution.

Spawning potential ratios are not the only approach to determine biological reference points. Maximum sustainable yield can also be used to derive the reference points, but this requires a production function, typically a stock-recruitment relationship. This combination of per recruit and stock recruitment information to derive biological reference points is often referred to as the Sissenwine-Shepherd approach (Sissenwine and Shepherd, 1987). The Gulf of Maine cod estimates of stock and recruitment spanned too small a range to allow direct estimation of a curve. However, there are a number of alternatives which can be used to create this relationship. For example, if the  $F_{40\%}$  under  $M=0.2$  is assumed to be a valid proxy for  $F_{msy}$ , then a steepness value of a Beverton-Holt stock-recruitment curve can be estimated which produces  $F_{msy}$  equal to the  $F_{40\%}$ . In this case, the steepness estimate is 0.69, resulting in  $F_{msy}$  of 0.18. The unfished recruitment,  $R_0$ , can then be determined either to produce the same  $SSB_{msy}$  as the  $F_{40\%}$  proxy, or else through a fitting exercise to minimize the residuals. The former approach was taken here, because the interest was more in changes in the  $F_{target}$  than changes in recruitment at much higher biomasses.

When the stock recruitment curve is set based on  $M=0.2$ , as described in the previous paragraph, but  $M$  increases to  $0.4$ , the  $F_{msy}$  value decreases from  $0.18$  to  $0.09$ . This is because the increase in natural mortality rate decreases the unexploited spawning stock biomass, resulting in a stock recruitment curve that effectively has a lower steepness. This occurs because the shape of the Beverton-Holt curve is steeper at smaller spawning stock sizes and the slope decreases monotonically. In the Gulf of Maine cod case, if  $M$  were to increase to  $0.8$  or  $1.6$ , then the unexploited stock recruitment line would no longer intersect the stock recruitment curve in the positive quadrant, meaning there is no sustainable yield under these conditions. Thus, the maximum sustainable yield approach reduces the fishing mortality rate when  $M$  increases within an assessment, the opposite change from the yield per recruit approach described above.

The different approaches to setting the  $F_{target}$  have short term as well as equilibrium impacts. Most importantly, the change in  $F_{target}$  will change the short term catch advice, with higher  $F$  resulting in higher quotas in the short term. This directional change has caused some to argue for use of the yield per recruit approach because it will increase the quota relative to the MSY approach of reducing the  $F$  when  $M$  increases. Since it is generally accepted that short term projections should use the most recent estimate of  $M$ , along with the recent selectivity, weights at age, and fecundity, the change in  $F_{target}$  has an immediate directional impact, with higher  $F_{target}$  resulting in higher quotas next year (Fig. 4). However, these short term gains in yield can be offset by reduced population size if future recruitment declines due to the reduced population size (Fig. 4). The location of the MSY value relative to the 15 year projection using the stock recruitment relationship demonstrates that 15 years is not sufficient to reach equilibrium in these deterministic projections. Equilibrium yield is even lower than the 15 year line for the stock recruitment relationship when  $F$  is above  $F_{msy}$ . Thus, the short term gains associated with high  $F_{target}$  values should be considered in light of potential losses in future yield if the high total mortality rate leads to lower recruitment in response to reductions in SSB.

### **Georges Bank Yellowtail Flounder Example**

The 2012 assessment for Georges Bank yellowtail flounder conducted a thorough examination of the timing and magnitude of change in the natural mortality rate needed to eliminate the retrospective pattern (Legault et al., 2012). The search found that a number of combinations could essentially eliminate the retrospective pattern and the one selected for demonstration purposes used  $M=0.2$  for years 1973-2004 and  $M=0.9$  for years 2005-2011. This  $M$  time series was applied to the data from the 2013 assessment (Legault et al., 2013) for this demonstration, extending the high  $M$  to 2012. This example demonstration is not meant to replace the accepted assessment, it is provided for demonstration purposes only.

When applied to the 2013 assessment data, the retrospective pattern remained small (SSB rho = -5% and  $F$  rho = -9%). This large change in the natural mortality rate had a large impact on the recruitment estimates in recent years (Fig. 5). The stock recruitment curve from the example assessment shows the typical scattered relationship (Fig. 6). Plotting replacement lines

associated with M values ranging from 0.2 to 0.9 shows most of the estimates stock and recruitment pairs above the low M replacement line and below the high M replacement line (Fig. 7). The location of the stock and recruitment pairs relative to the replacement lines cannot be used to determine appropriate spawning potential ratio proxy for Fmsy (Legault and Brooks, 2013), but are important when used with stock recruitment curves to determine equilibrium conditions. A range of Beverton-Holt stock recruitment curves were fit to the stock and recruitment pairs with nearly equal fits (Table 4, Fig. 8). The Beverton Holt SR curve was defined as  $R = \alpha * SSB / (\beta + SSB)$ . Combining the stock recruitment curves with the range of M values allows estimation of the steepness associated with each SR curve and M as well as direct estimates of Fmsy values (Table 5). Note that a number of the SR curves do not intersect with replacement lines associated with high M values, thus resulting in no equilibrium solutions for MSY reference points.

As can be seen in Table 5, the curves were fit by fixing M=0.2 and steepness at values from 0.65 to 0.9 in steps of 0.05 and finding the unfished recruitment that minimized the residual sum of squares. These curves were then converted to the alpha and beta version of the Beverton Holt curve defined above and held fixed while M was allowed to change. Many other curves could have been fit to these stock and recruitment pairs with nearly equal fits. This approach was chosen to allow a simple ordering of the SR curves. The steepness values associated with a given SR curve decrease as M increases (Fig. 9) while the SR curves increase in steepness for a given M. Thus, the use of the steepness formulation for the Beverton Holt curve is problematic when M changes within an assessment because a given SR curve has different steepness values for differing M values. Application of meta-analysis results for similar stocks will be difficult as well. Conceptually, holding the SR curve fixed when M changes is similar to the standard fisheries approach of fitting a SR curve to estimated stock-recruit pairs when fishing mortality varies throughout the time series.

For each stock recruitment curve and natural mortality rate combination, there is one Fmsy which maximizes the equilibrium yield. This was found through a search over values of F from zero to two in steps of 0.01. The equilibrium yield was computed from the yield per recruit and intersection of the replacement line with that total mortality rate (natural plus fishing) with the SR curve for each F value. For a given SR curve, the Fmsy values can initially increase as M increases, but eventually will decrease to zero with a high enough M as the replacement line moves to the left of the SR curve (Fig. 10). For a given M, higher steepness in the SR curve results in higher Fmsy, as expected. The associated MSY values decrease with increasing M, with SR curves 1 and 2 in combination with low M producing extremely high values (Fig. 11). Similarly, the Bmsy values decrease with increasing M (Fig. 12). All of the Bmsy values for M above 0.6 are well below historical catch amounts for this stock. Thus, using the most recent M in the example assessment of 0.9 results in low Bmsy and MSY regardless of which SR curve is used (some of the SR curves do not have equilibrium values because the replacement line for M=0.9 is to the left of the curves).

The ratio of Bmsy to unfished spawning stock biomass (Bmsy/B0) is surprisingly consistent, increasing with increasing M until the replacement line just barely intersects the SR curve

( $M=0.9$  and SR curve 4; Fig. 13). These are not direct estimates of spawning potential ratios from per recruit analysis because they include the changes in recruitment associated with the SR curves. However, they do indicate that  $F_{msy}$  is generally associated with reductions in the spawning stock biomass to between 25% and 45% of unfished conditions over a range of SR curves and  $M$  values for this example.

Cadrin (WP 26) recommends comparing the equilibrium expectations with historical productivity. These equilibrium values for a range of  $F$  between zero and two are compared to the observed values of spawning stock biomass, fishery yield, and fishing mortality rate (Fig. 14). These plots demonstrate the problem with this recommendation when  $M$  changes within an assessment. Despite the SR curves having quite similar fits to the stock and recruitment pairs, the equilibrium lines associated with the curves vary dramatically. For SR curve 1, none of the equilibrium SSB or yield lines as a function of  $F$  follow the trajectory of the VPA SSB (denoted "Observed" based on Cadrin's notation). However, the  $M=0.3$ , 0.4 and 0.5 lines on the yield versus spawning stock biomass plots for SR curve 1 do pass through the observations. As the SR curves increase in steepness, the values of  $M$  which pass through the SSB and yield as a function of  $F$  observations increase. In the yield versus spawning stock biomass plots, the equilibrium curves are only plotted for values of  $F$  up to 2.0, demonstrating the higher  $F$  are needed for many of the curves to pass through the observations. Given that  $M$  has substantially increased from 0.2 to 0.9 in this example, it is unclear to us how to use these figures to help determine which  $F_{msy}$  value is appropriate as none of the equilibrium curves with  $M=0.9$  pass through the middle of the observations. We do not think the equilibrium lines should be expected to pass through the middle of the observations because the observations were mostly derived from  $M=0.2$  conditions. This highlights yet another difficulty with allowing  $M$  to change dramatically within an assessment, the relationships between historical and current productivity are not easy to interpret especially under equilibrium conditions.

## Discussion

Both the yield per recruit approach, which increases the  $F_{target}$  as  $M$  increases, and the maximum sustainable yield approach, which eventually decreases the  $F_{target}$  as  $M$  increases, can result in extreme  $F_{targets}$  when  $M$  increases substantially. Fishing targets of  $>2$  or  $<0.01$  are not reasonable for long-lived gadoids like cod. The fact that these two extremes are encountered under the same high  $M$  condition, but depend only on the approach used to define the reference point, means that neither approach is obviously more correct than the other. It is hard to understand how  $F_{targets}$  could change so dramatically when  $M$  increases within an assessment because there is not enough time for evolution to occur and the species to adopt a new life history to account for this change in  $M$  (nor would there be a reason for the species to adapt if the change in  $M$  was strictly to address a different problem in the assessment causing a retrospective pattern). If instead, the change in  $M$  is considered as another fishing fleet which cannot be controlled, then perhaps the best a manager can do is to manage at the rate that would be appropriate were that fleet to go away, meaning fish at the rate appropriate for the life history of the species (denoted  $F_{con}$  in Fig. 4).

In some parts of the world, natural mortality is held constant over time in stock assessments despite estimates of changes due to predation by other species. For example, stock assessments conducted by the Alaska Fisheries Science Center hold  $M$  constant over time in all their single species stock assessments. These single species stock assessments are used to set the biological reference points which serve as the starting point for catch advice. Multispecies stock assessments have been conducted which provide time and age-varying estimates of  $M$  for some species (e.g., Livingston and Methot, 1998; Hollowed et al., 2000; Jurado-Molina et al., 2005). These models generally provide some ecosystem considerations that have functionally served to evaluate the level of precaution built into the catch advice arising from the single species stock assessments. This approach avoids the difficulty of determining biological reference points when natural mortality changes over time within an assessment.

There are a number of rules of thumb in fisheries that describe how to act generally. One such rule is that  $F_{\text{target}}=M$  is an appropriate fishing mortality rate. This is generally true over a wide range of life histories. However, it encounters difficulty when  $M$  is changing within an assessment because the ability to estimate  $M$  and the non-stationarity of  $M$  mean that predicting the correct  $F_{\text{target}}$  will be challenging. There are also other rules of thumb related to  $M$ . For example,  $M/K$  is generally considered to be a constant value for a species. Under this rule, an increasing  $M$  within an assessment should be associated with an increased growth rate of the species. Similarly, a species with a given  $M$  should have a fecundity strategy that allows each adult to replace itself. So when  $M$  increases within an assessment, either the fecundity of younger fish or the survival of the eggs produced by these younger fish should increase (assuming  $F$  is constant). Finally,  $M$  is often related to maximum observed age in a population. An increase in  $M$  should reduce the maximum observed age in the population (again, assuming  $F$  is constant). In the case of Gulf of Maine cod, no substantial changes have been observed in growth, maturation or maximum age in the population (NEFSC, 2013). The absence of life history trait changes can confound the interpretation of assumed changes in  $M$  within a stock assessment. Thus, rules of thumb are useful guides but must be kept in context of their origin before they can be applied to the situation of an increasing  $M$  within an assessment.

### **Recommendations**

We recommend using natural mortality rates that do not change over time. This avoids the difficulty of estimating how much this parameter changes over time and the difficulty of setting biological reference points which vary depending on the approach taken to address the change in natural mortality. If the natural mortality rate is allowed to change over time within an assessment, we recommend basing the biological reference points on the natural mortality rate considered most appropriate for the life history of the species. This avoids large scale changes in the reference points as transient changes in the natural mortality rate occur, providing for more stable management.

### **Acknowledgements**



We thank Jim Ianelli for providing a perspective from the Alaska Fisheries Science Center and Grant Thompson for discussions about the possibility of  $F_{msy}$  increasing when  $M$  increases.

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Table 1. Life history and fishery parameters used in calculations (average of values for years 2009-2011). Weights are in kg. Time of spawning for SSB calculations is 0.25 of the year.

Age	Catch Weight	Stock Weight	Maturity	Selectivity
1	0.310	0.119	0.092	0.004
2	1.015	0.520	0.287	0.027
3	2.068	1.256	0.613	0.165
4	3.068	2.194	0.862	0.588
5	3.786	3.123	0.961	0.911
6	4.551	3.819	0.990	0.987
7	5.795	4.767	0.997	0.998
8	7.561	6.546	0.999	1.000
9+	12.494	12.495	1.000	1.000

Table 2. Biological reference points associated with F40% for a range of M values and the age vectors of biological and fishery characteristics in Table 1. SSBPR denotes spawning stock biomass per recruit, YPR denotes yield per recruit, and R, SSB, and Yield denote equilibrium deterministic recruitment, spawning stock biomass, and yield, respectively. Units for SSBPR, YPR, recruitment, SSB, and Yield are kg, kg, thousands of fish, metric tons, and metric tons, respectively. Note that F40% hit the arbitrary upper bound of 10 for M=1.6 (the spawning potential ratio was 57% instead of the desired 40%).

	M=0.2	M=0.4	M=0.8	M=1.6
SSBPR(F=0)	20.344	4.088	0.571	0.064
F40%	0.183	0.452	2.729	10
SSBPR(F40%)	8.138	1.635	0.228	0.036
YPR(F40%)	1.400	0.570	0.236	0.063
R(F40%)	10214	10214	10214	10214
SSB(F40%)	83119	16702	2333	367
Yield(F40%)	14299	5820	2410	645

Table 3. Equilibrium Jan-1 population abundance at age (thousands of fish) under two fishing conditions and four values of natural mortality for the Gulf of Maine cod example.

Age	F=0				F40%			
	M=0.2	M=0.4	M=0.8	M=1.6	M=0.2	M=0.4	M=0.8	M=1.6
1	10214.0	10214.0	10214.0	10214.0	10214.0	10214.0	10214.0	10214.0
2	8362.5	6846.6	4589.4	2062.2	8356.4	6834.3	4539.6	1981.3
3	6846.6	4589.4	2062.2	416.3	6807.9	4525.6	1894.9	305.4
4	5605.6	3076.4	926.6	84.1	5408.2	2815.6	542.7	11.8
5	4589.4	2062.2	416.3	17.0	3976.4	1446.9	49.0	0.0
6	3757.5	1382.3	187.1	3.4	2755.9	642.6	1.8	0.0
7	3076.4	926.6	84.1	0.7	1883.7	275.7	0.1	0.0
8	2518.7	621.1	37.8	0.1	1284.9	117.7	0.0	0.0
9+	11376.3	1262.9	30.8	0.0	2754.3	87.6	0.0	0.0

Table 4. Parameters of the Beverton Holt stock recruitment relationship and residual sum of squares for six different curves fit to the Georges Bank yellowtail flounder example.

SR Curve	alpha	beta	RSS
1	1040000.00	360898.99	17.81
2	268800.00	74242.08	16.36
3	100363.64	21560.20	16.14
4	61866.67	9967.69	16.61
5	44984.62	5116.04	17.70
6	34971.43	2504.20	19.35

Table 5. Estimates of steepness, unfished SSB (B0), unfished recruitment (R0), Fmsy, Bmsy, Rmsy, and MSY for all 48 combinations of the the six SR curves and eight natural mortality rates (M) for the Georges Bank yellowtail flounder example.

SR Curve	M	steepness	B0	R0	Fmsy	Bmsy	Rmsy	MSY
1	0.2	0.650	2320065	900000	0.31	733302	696978	162737
2	0.2	0.700	618684	240000	0.36	186127	192154	46526
3	0.2	0.750	237162	92000	0.41	68733	76399	18991
4	0.2	0.800	149515	58000	0.48	40837	49729	12682
5	0.2	0.850	110848	43000	0.56	28641	38167	9921
6	0.2	0.900	87647	34000	0.67	21129	31266	8255
1	0.3	0.498	1069794	777655	0.31	392542	541839	80557
2	0.3	0.555	295537	214832	0.38	103939	156800	25070
3	0.3	0.616	116507	84691	0.48	38187	64147	10984
4	0.3	0.681	75140	54621	0.60	23153	43248	7800
5	0.3	0.751	56768	41266	0.78	16115	34145	6451
6	0.3	0.828	45605	33151	1.11	11475	28707	5665
1	0.4	0.377	512589	610303	0.26	209902	382442	34352
2	0.4	0.432	151521	180405	0.35	59463	119544	12411
3	0.4	0.494	62734	74693	0.48	22889	51682	6089
4	0.4	0.566	41994	49999	0.65	14358	36516	4737
5	0.4	0.649	32666	38893	0.96	10002	29762	4240
6	0.4	0.746	26868	31990	1.66	6968	25726	4017
1	0.5	0.286	216685	390164	0.17	96178	218837	10007
2	0.5	0.335	75041	135120	0.27	32355	81588	5029
3	0.5	0.393	34179	61542	0.42	13642	38894	3029
4	0.5	0.463	24391	43919	0.63	9099	29523	2724
5	0.5	0.550	19867	35773	1.03	6606	25351	2730
6	0.5	0.660	16918	30462	2.02	4721	22851	2855
1	0.6	0.218	42367	109263	0.05	18721	51287	567
2	0.6	0.260	29987	77334	0.15	14245	43272	1214
3	0.6	0.311	17356	44761	0.30	7673	26343	1197
4	0.6	0.376	14021	36161	0.54	5652	22386	1402
5	0.6	0.460	12327	31791	0.98	4460	20953	1669
6	0.6	0.575	11056	28513	2.14	3362	20041	1990
1	0.7	NA	NA	NA	0.00	NA	NA	0
2	0.7	0.203	1505	5341	0.01	764	2740	4
3	0.7	0.247	6722	23854	0.16	3116	12673	258
4	0.7	0.304	7466	26495	0.39	3252	15217	579
5	0.7	0.383	7561	26830	0.83	2977	16549	931
6	0.7	0.496	7351	26085	2.05	2436	17245	1338
1	0.8	NA	NA	NA	0.00	NA	NA	0
2	0.8	NA	NA	NA	0.00	NA	NA	0

3	0.8	NA	NA	NA	0.00	NA	NA	0
4	0.8	0.247	3101	14679	0.21	1426	7742	138
5	0.8	0.317	4386	20765	0.62	1866	12025	440
6	0.8	0.424	4883	23116	1.80	1759	14428	849
1	0.9	NA	NA	NA	0.00	NA	NA	0
2	0.9	NA	NA	NA	0.00	NA	NA	0
3	0.9	NA	NA	NA	0.00	NA	NA	0
4	0.9	0.201	73	450	0.01	11	70	0
5	0.9	0.263	2185	13462	0.38	991	7302	147
6	0.9	0.362	3172	19542	1.45	1234	11544	492

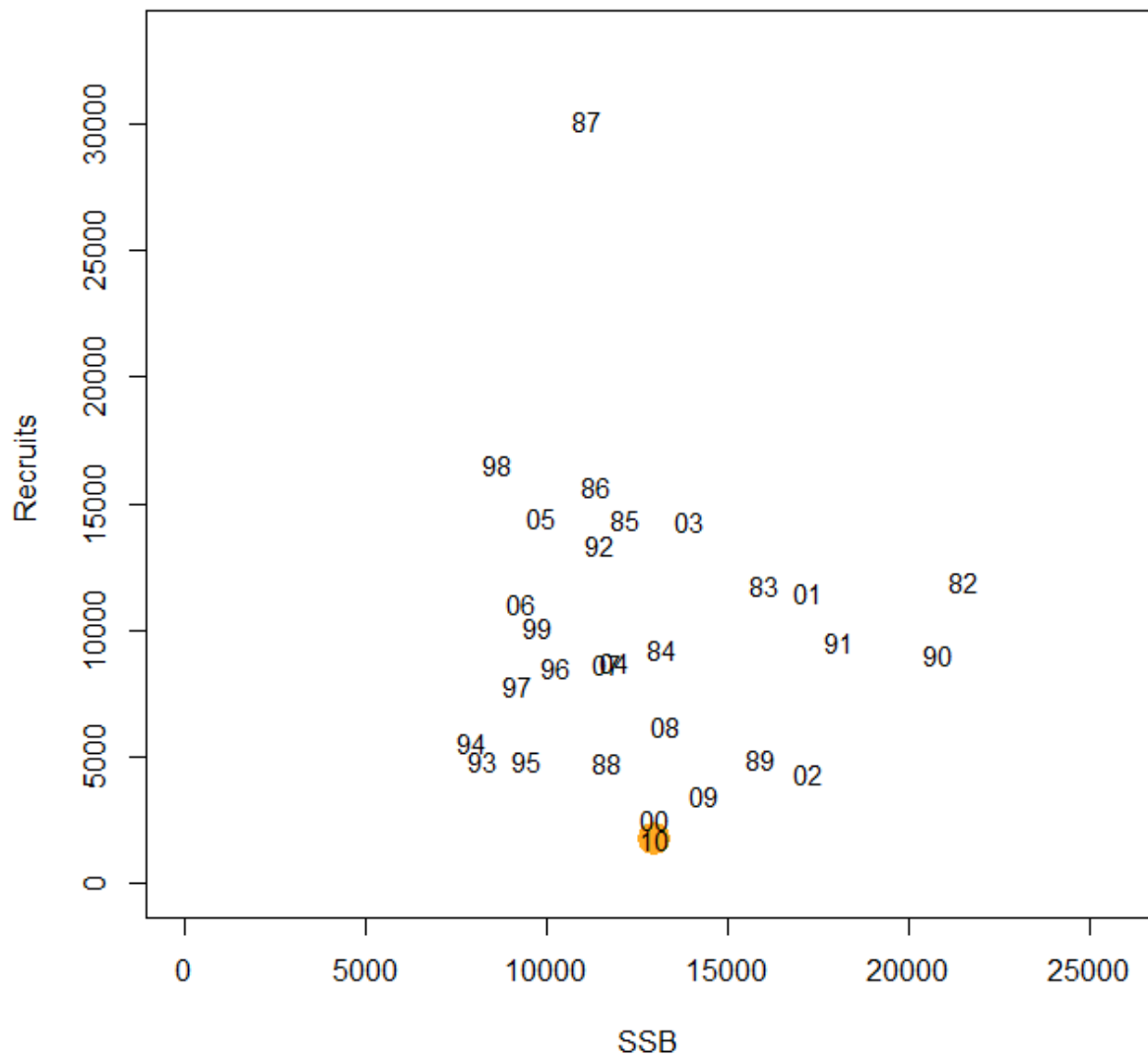


Figure 1. Stock recruitment relationship for the Gulf of Maine cod example. The two digit numbers denote the year of SSB and year-class of recruitment. Units are thousands of fish and metric tons for recruits and spawning stock biomass, respectively.



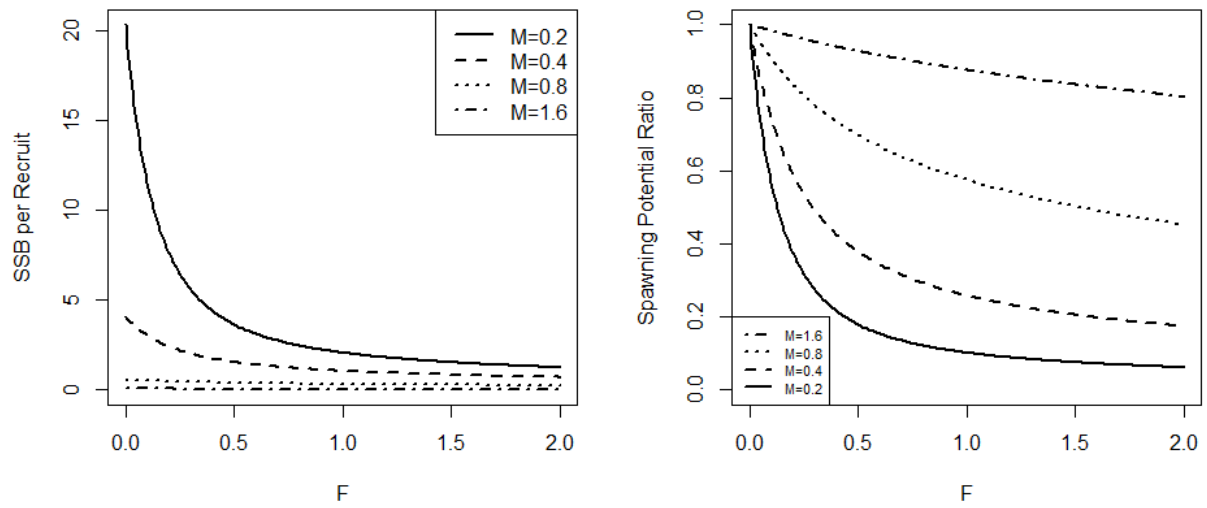


Figure 2. Spawning stock biomass per recruit (kg, left panel) and spawning potential ratio (right panel) as a function of the fishing mortality rate (F) for four natural mortality rates (M) for the Gulf of Maine cod example.

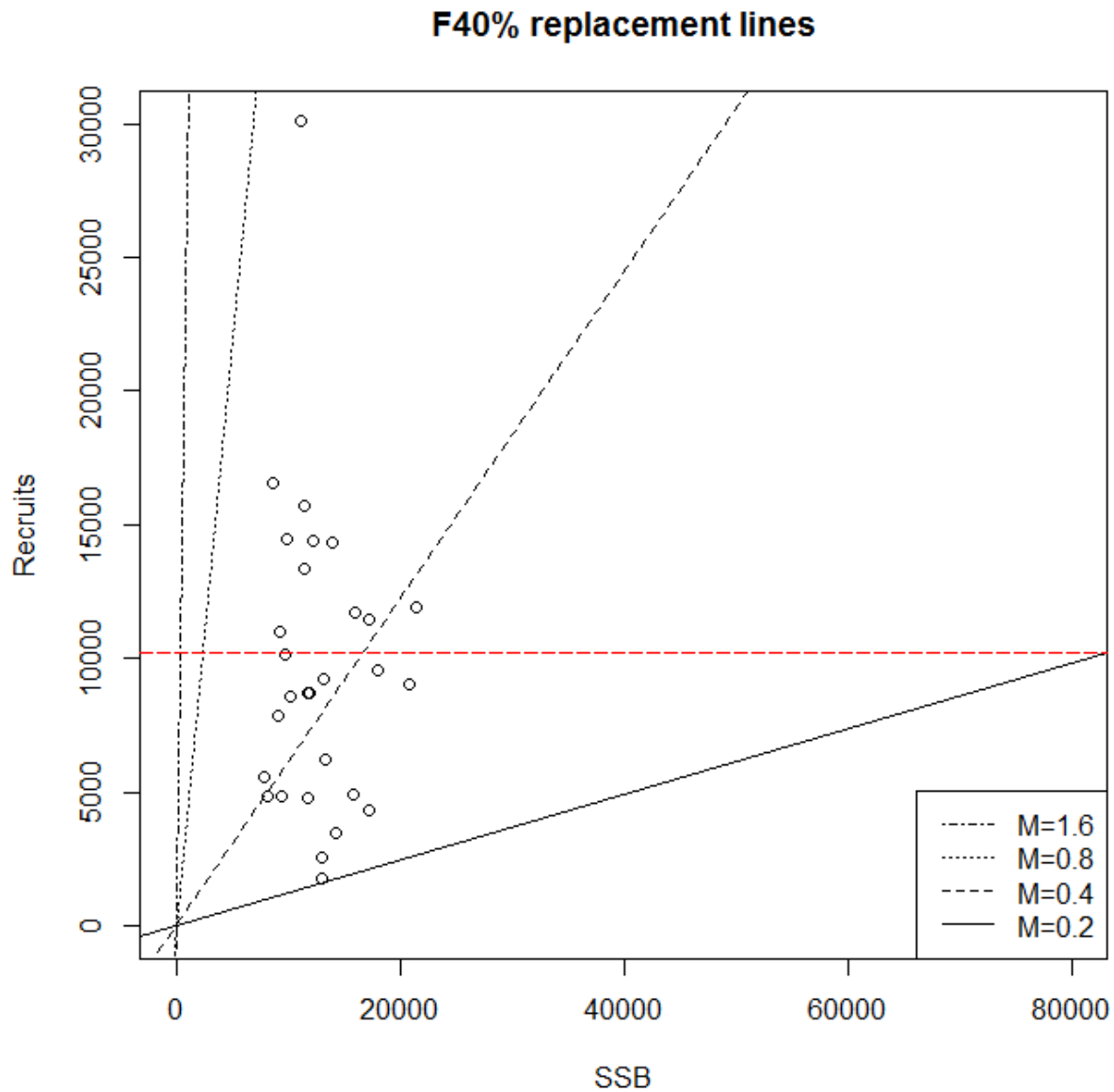


Figure 3. Replacement lines (lines with slope  $1/SPR$ ) associated with the F40% values for four natural mortality rates for the Gulf of Maine cod example. The horizontal line indicates the mean recruitment used in determining the biomass reference points. The intersection of the replacement lines with the horizontal line indicates the equilibrium spawning stock biomass and recruitment. The open circles denote the estimates of SSB and recruitment from the assessment.

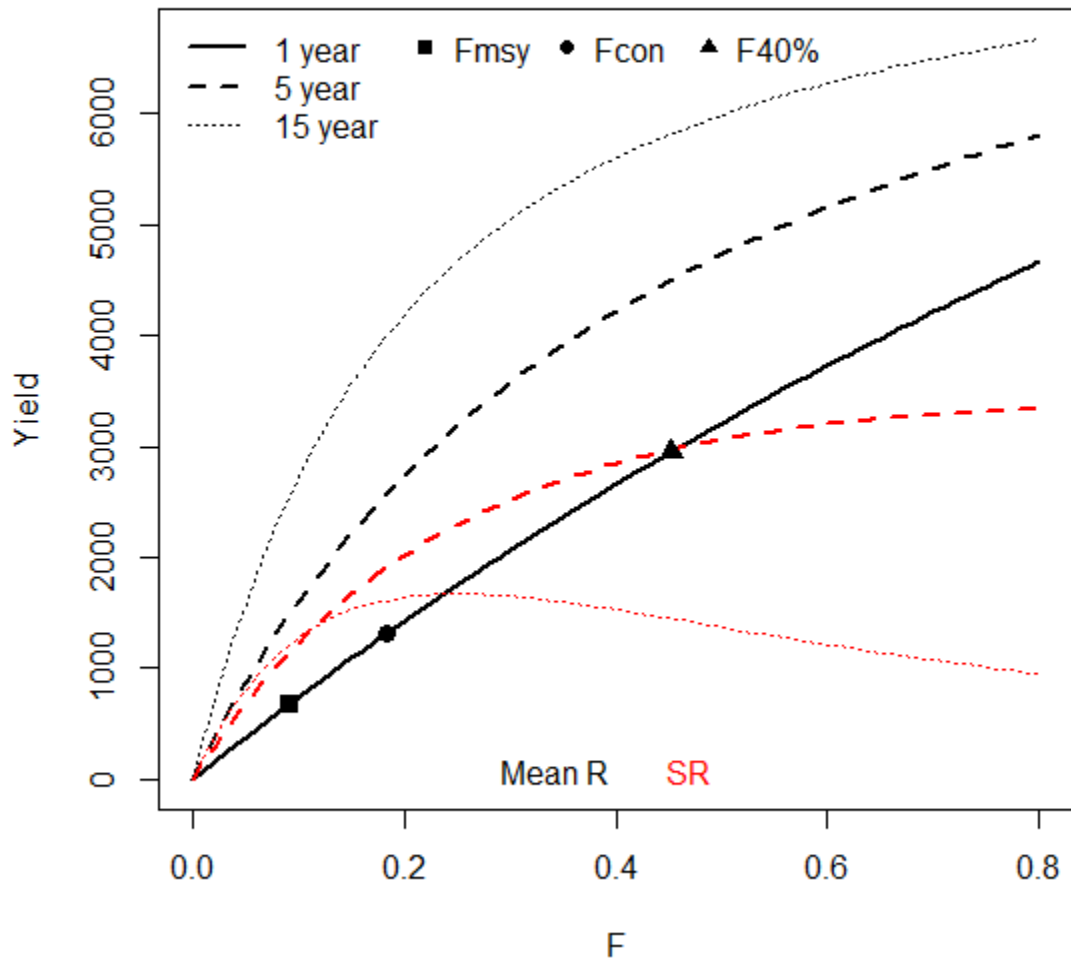


Figure 4. Yield as a function of the  $F_{target}$  when  $M$  remains at 0.4 in the future for the Gulf of Maine cod example. Three specific values of  $F_{target}$  are highlighted:  $F_{msy}$  and  $F_{40\%}$  are computed according to the increased  $M$  of 0.4 within the assessment time series, while  $F_{con}$  is computed assuming  $M=0.2$  and  $F_{40\%}$ . The solid line denotes the yield next year, the dashed lines denote the yield in 5 years, and the dotted lines denote the yield in 15 years assuming recruitment is either determined as a constant value of 10.214 million fish per year (black lines) or deterministically from the stock recruitment relationship described in the text (red lines). Note the next year projections are the same for the two recruitment scenarios.

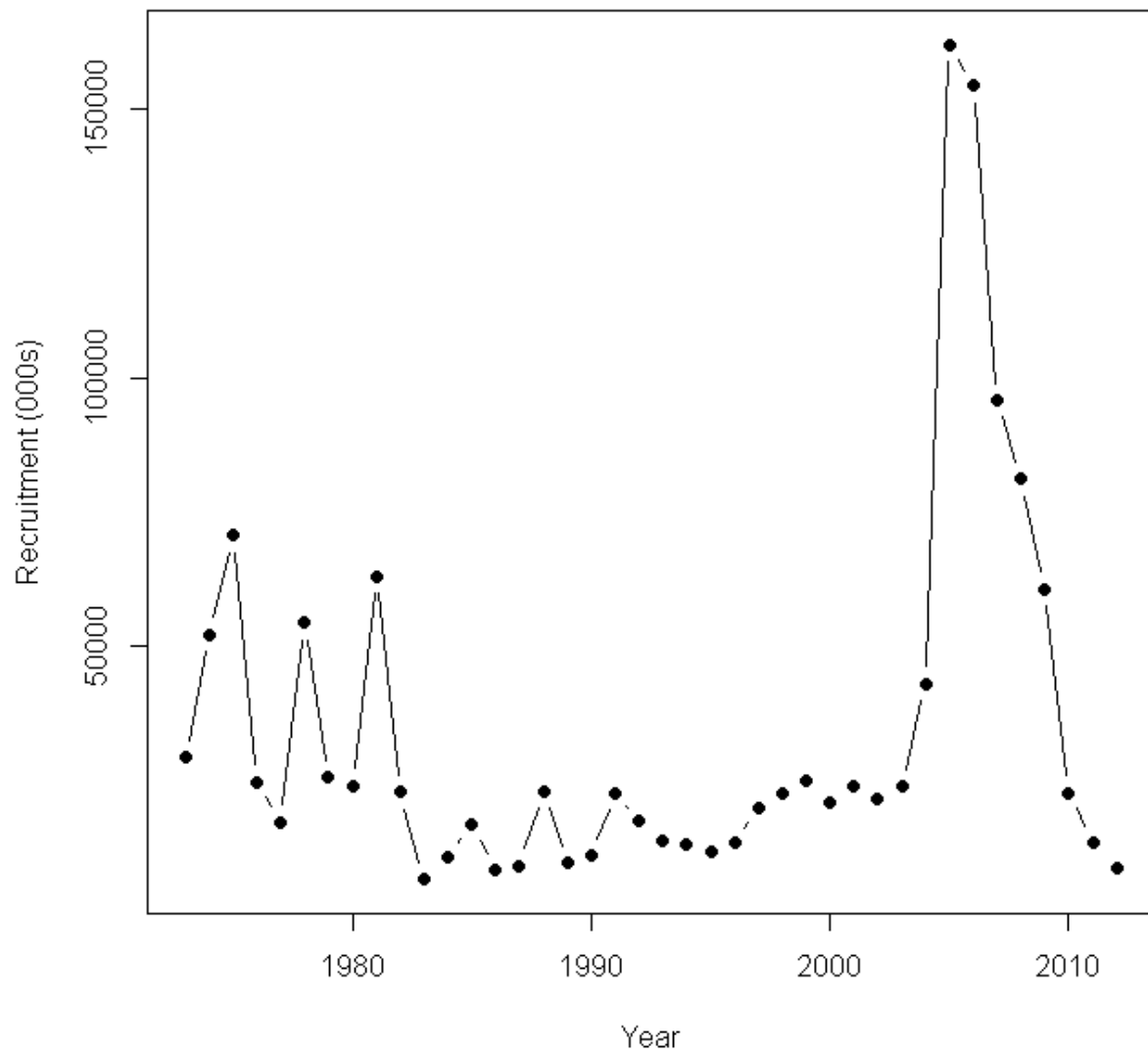


Figure 5. Estimated recruitment (000s of fish at age 1) for the Georges Bank yellowtail flounder example with sudden increase in  $M$  from 0.2 to 0.9 between 2004 and 2005.

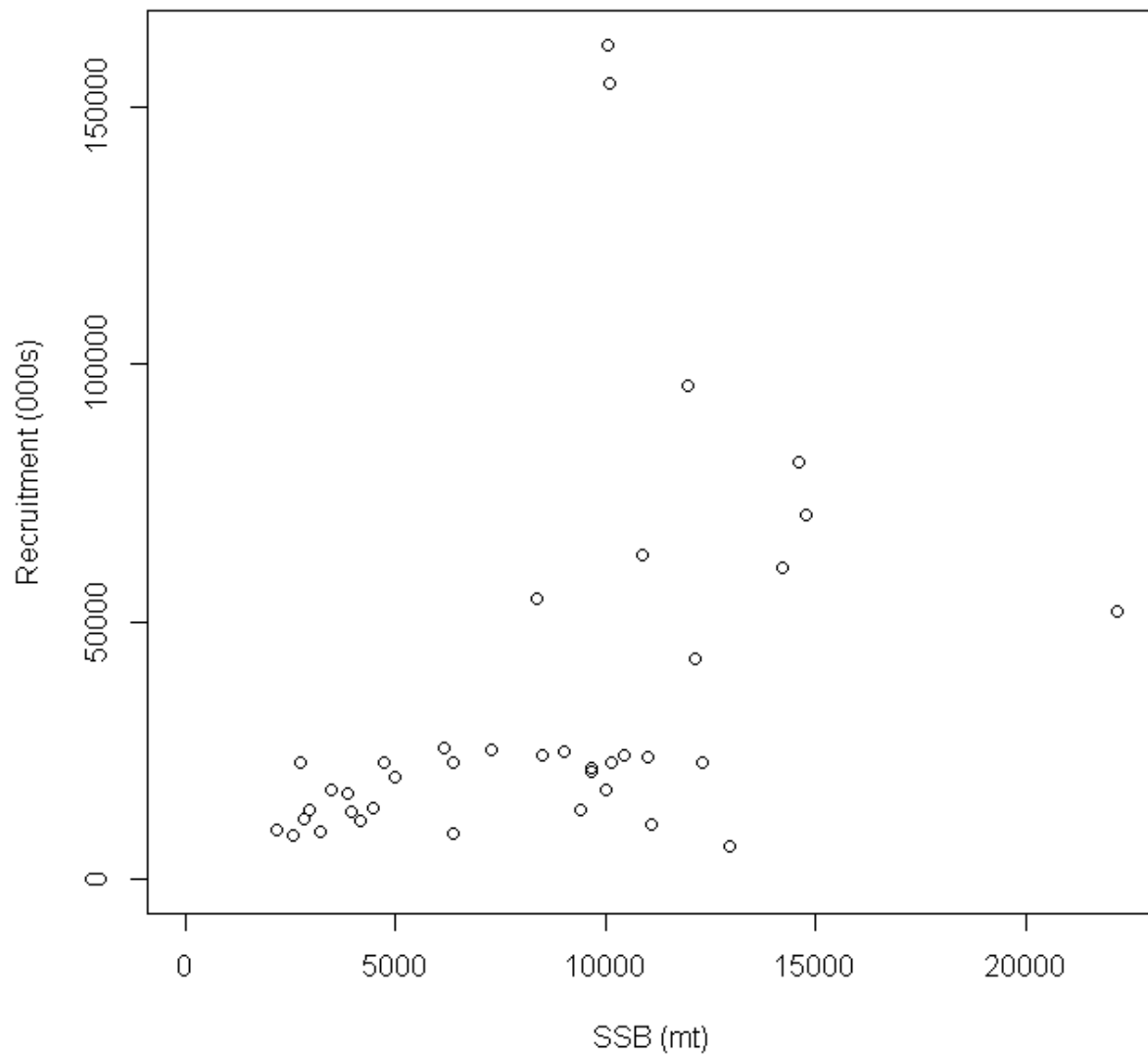


Figure 6. Stock and recruitment estimates for the Georges Bank yellowtail flounder example.

### Unfished Replacement Lines

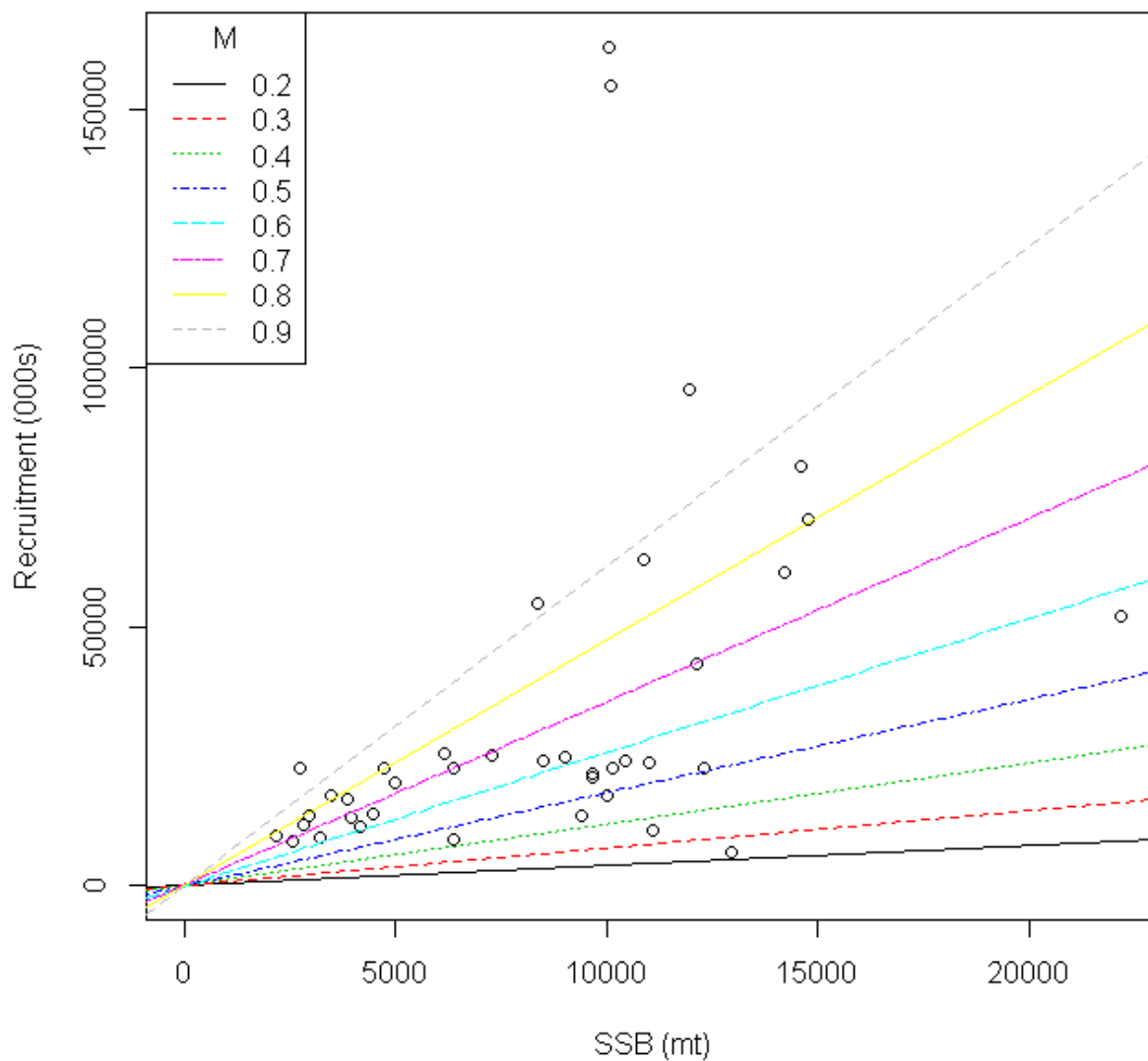


Figure 7. Stock and recruitment estimates along with unfished replacement lines for a range of natural mortality (M) values for the Georges Bank yellowtail flounder example.

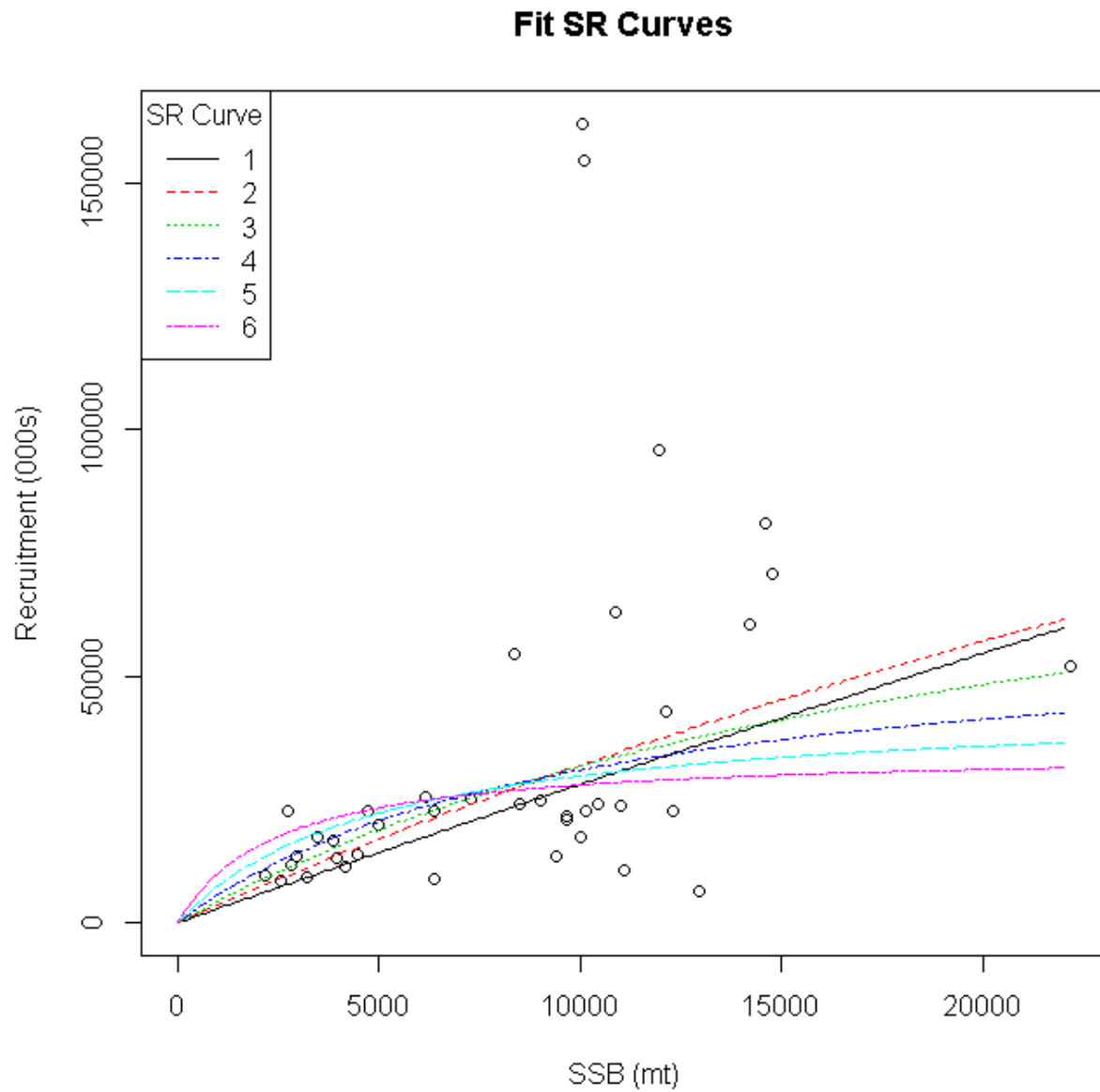


Figure 8. Stock and recruitment estimates along with six Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example. See Table 4 for the parameters associated with each curve.

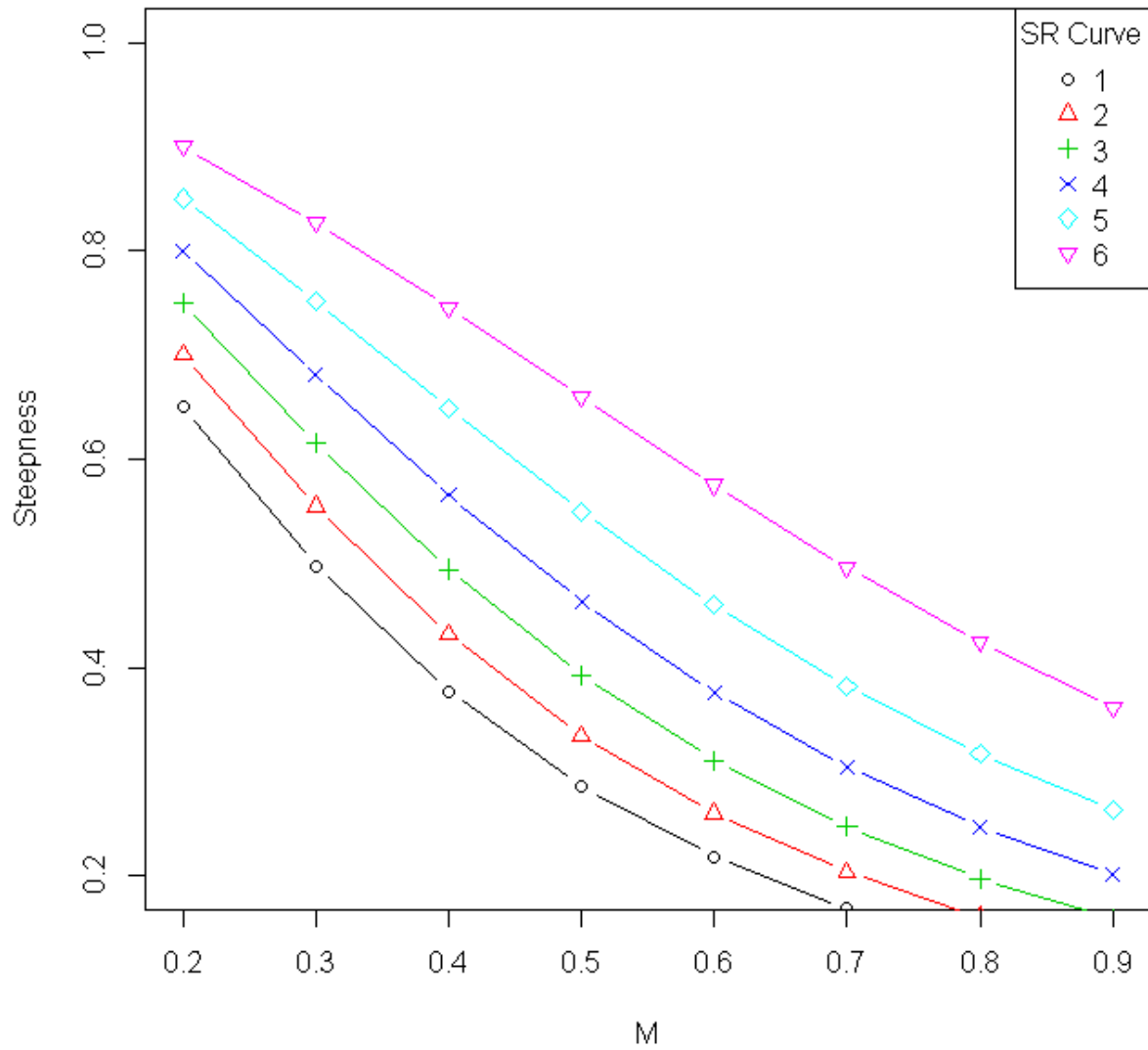


Figure 9. Steepness values associated with natural mortality rates (M) and Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example.



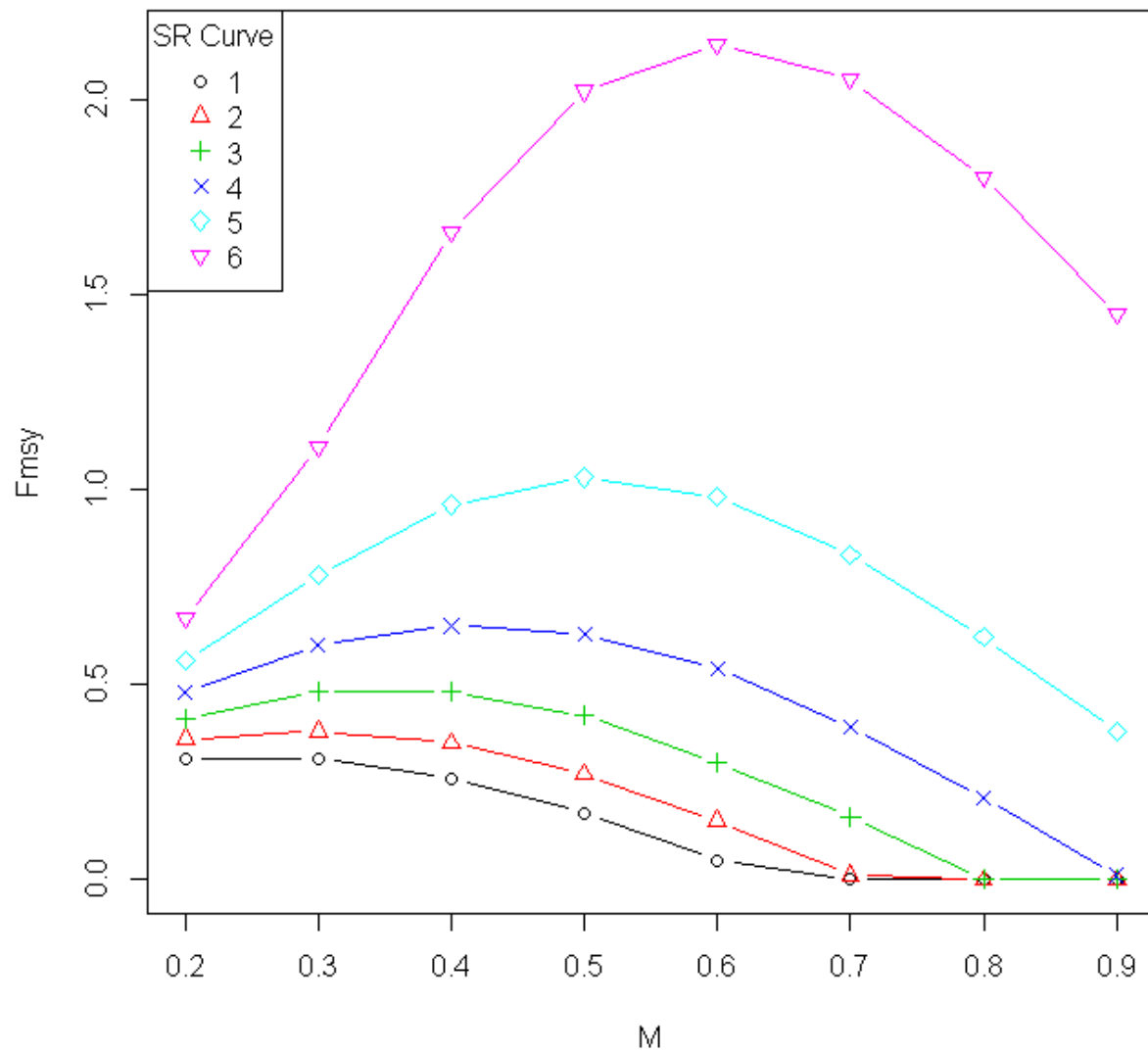


Figure 10. Fmsy values associated with natural mortality rates (M) and Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example.

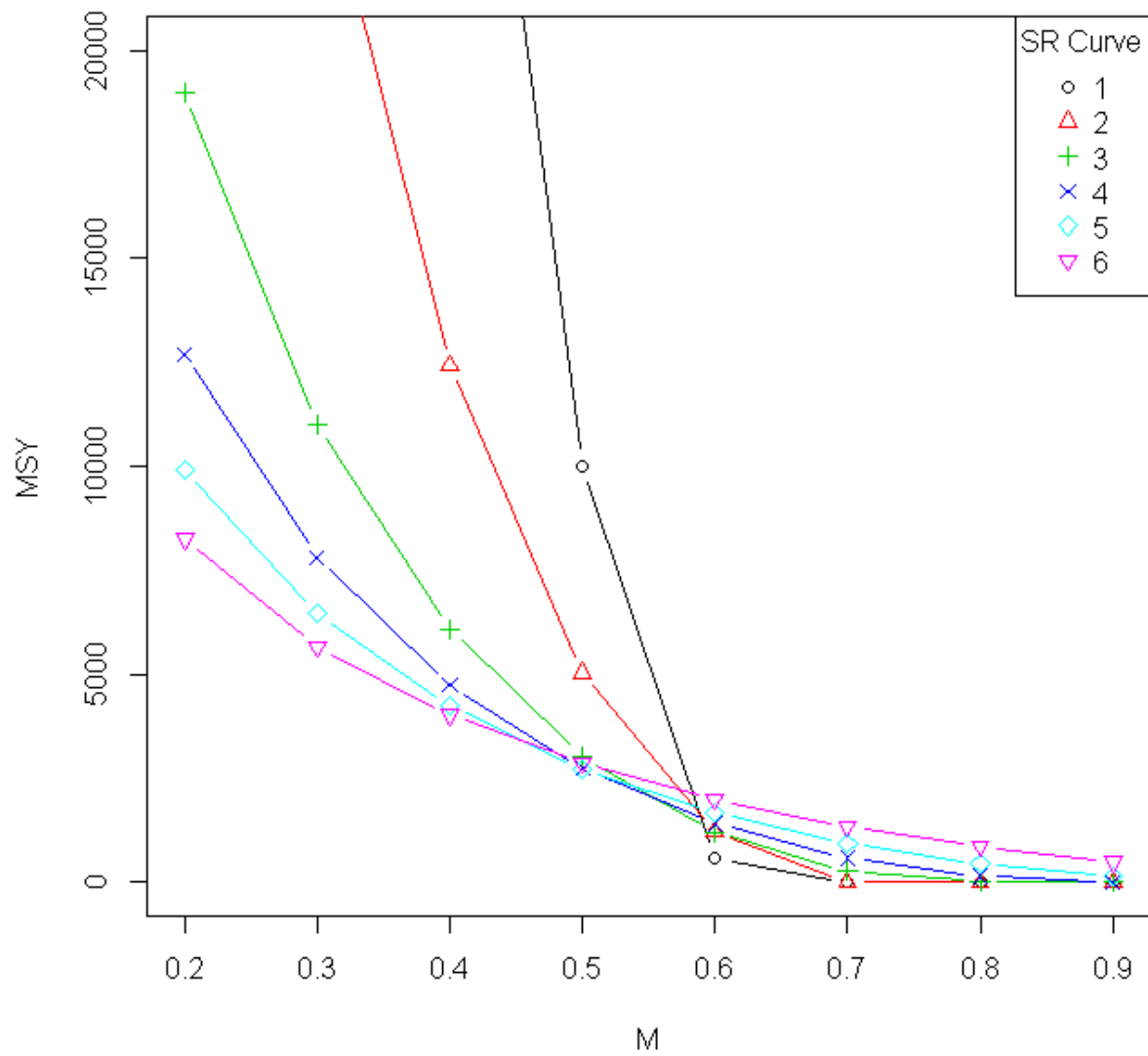


Figure 11. MSY values associated with natural mortality rates (M) and Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example.

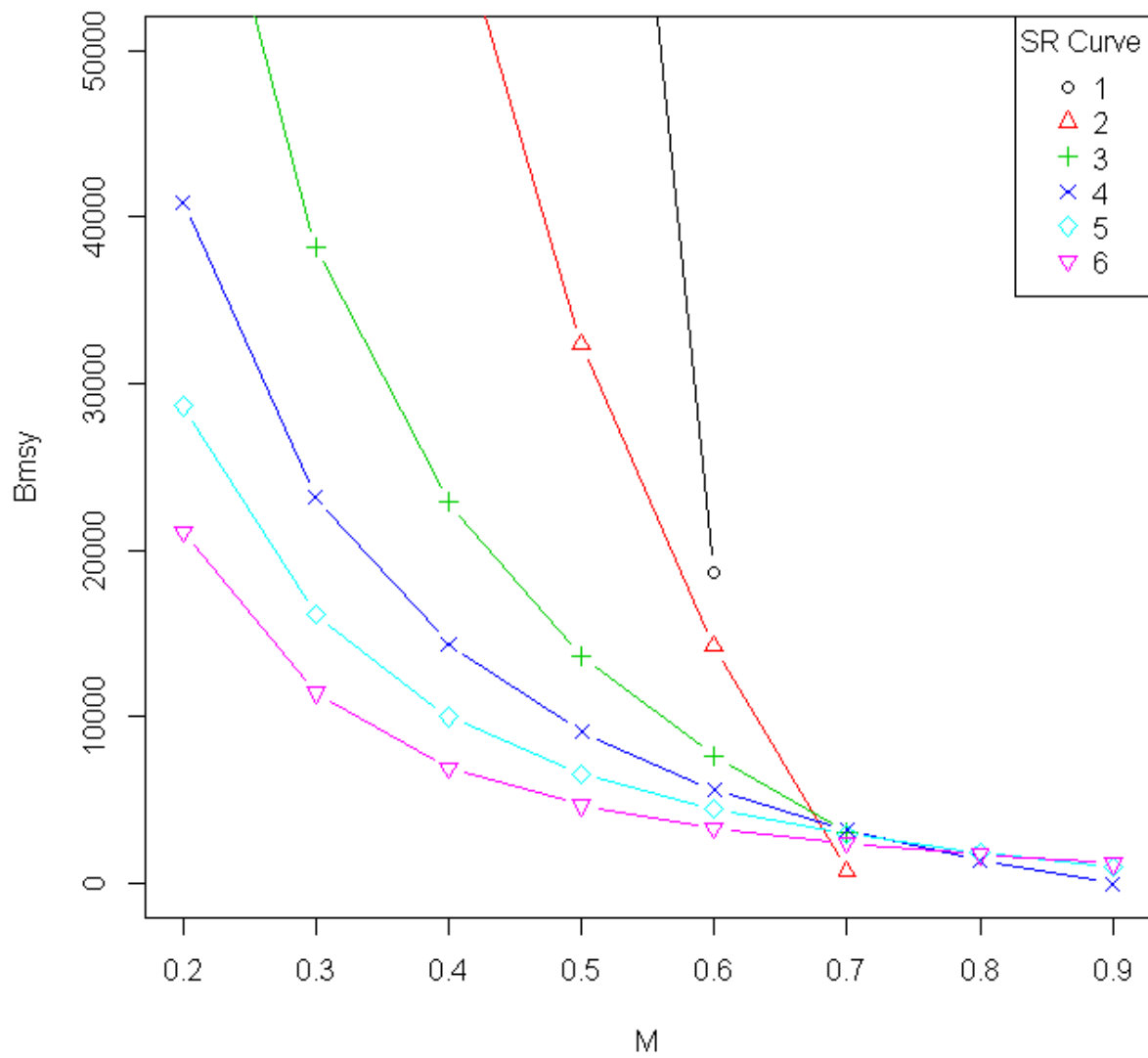


Figure 12. Bmsy values associated with natural mortality rates (M) and Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example.

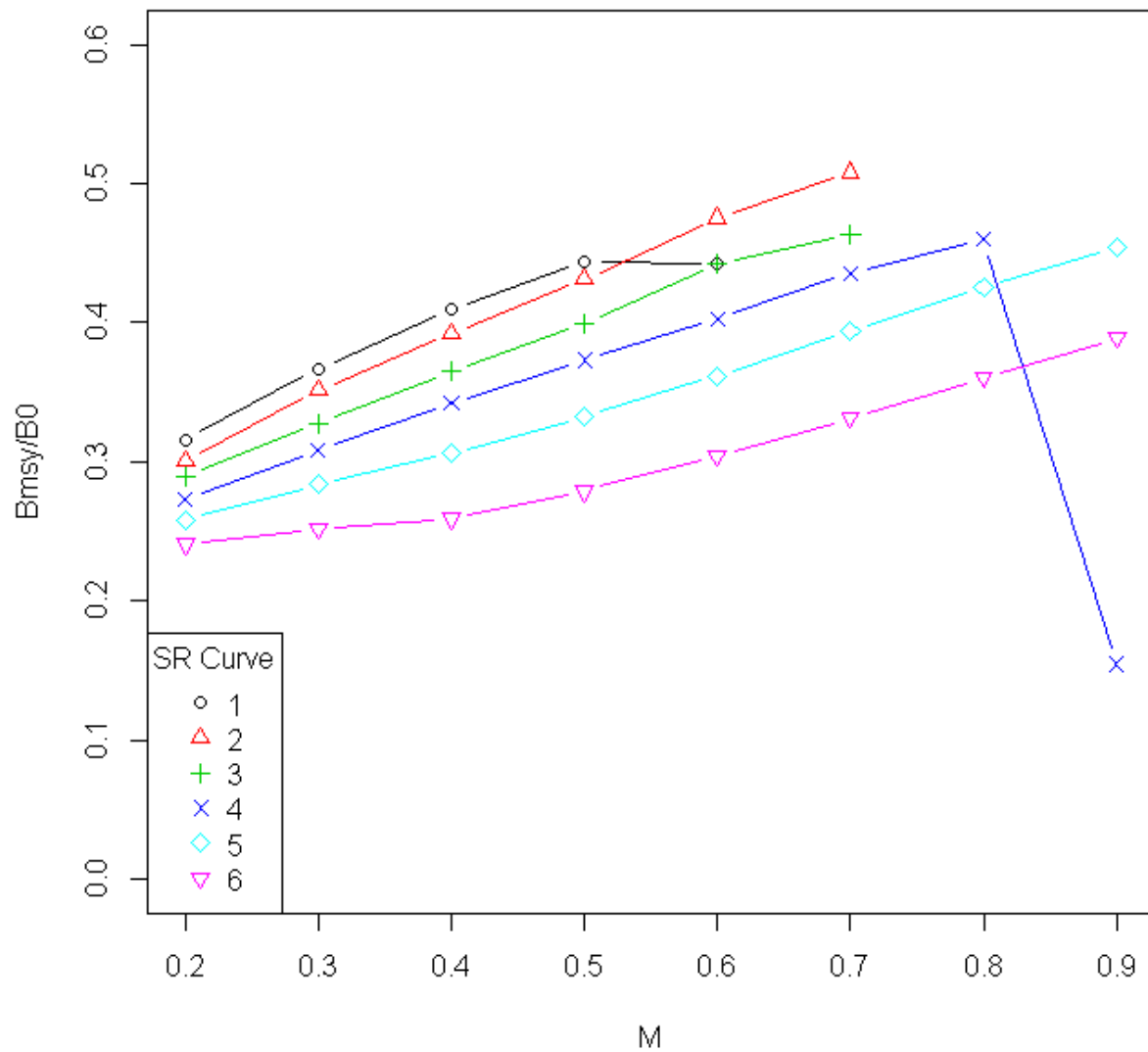
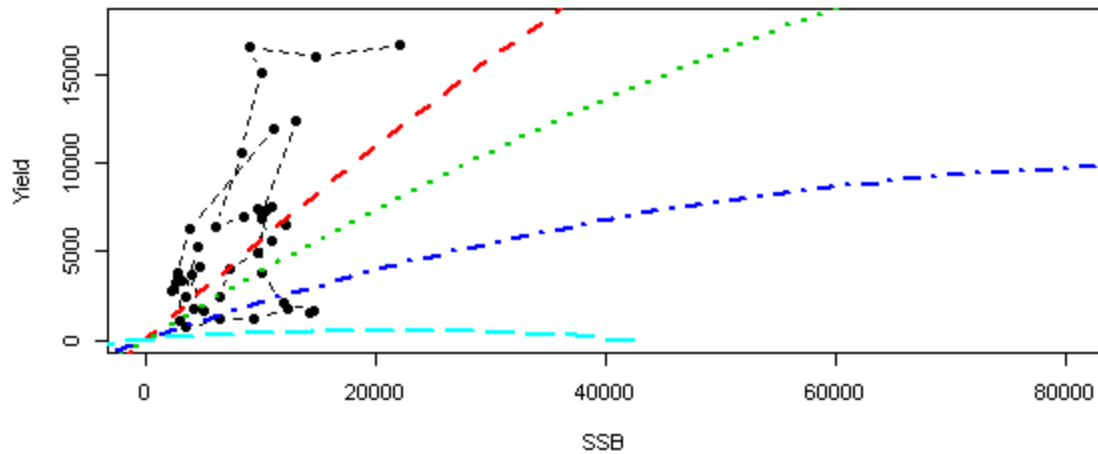
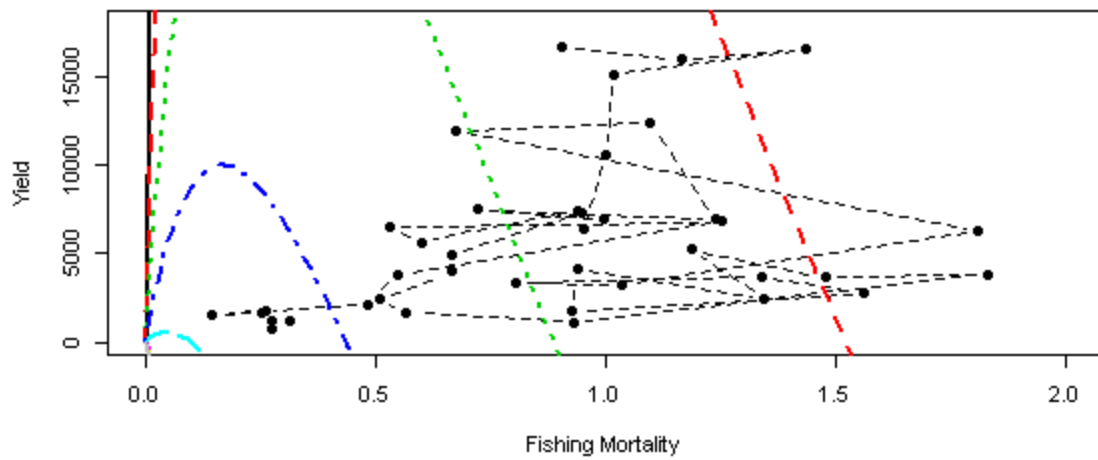
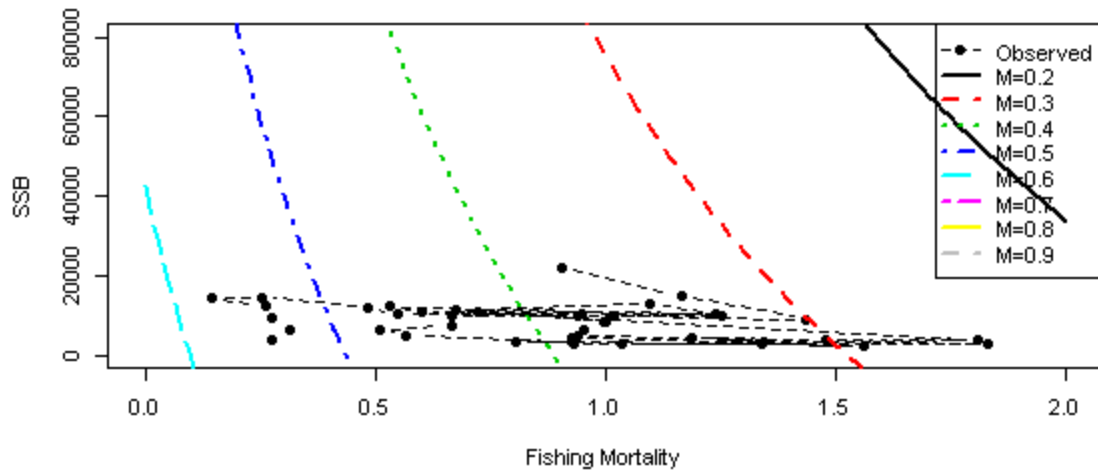
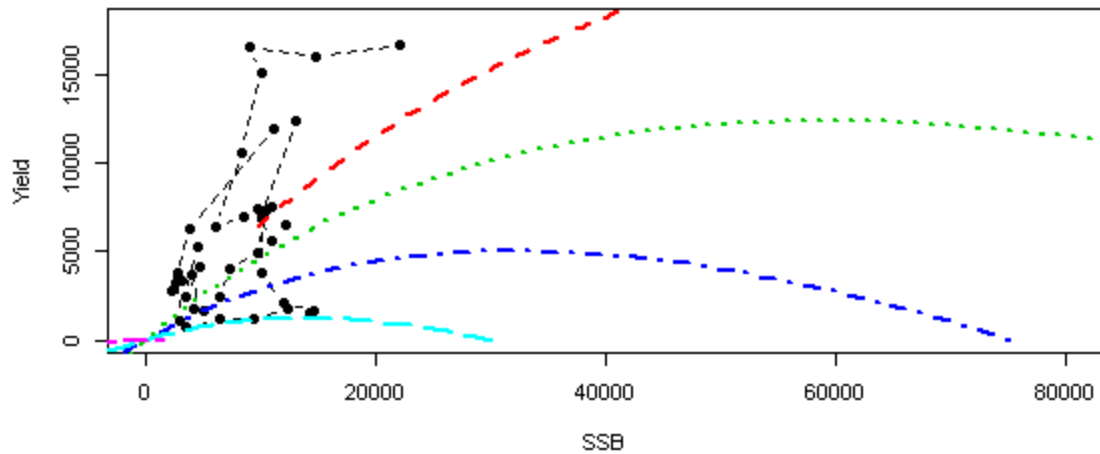
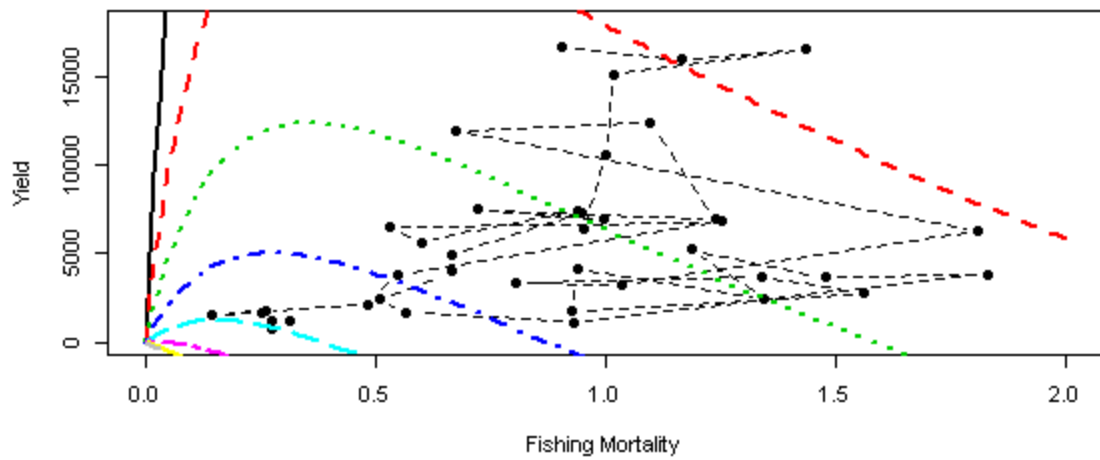
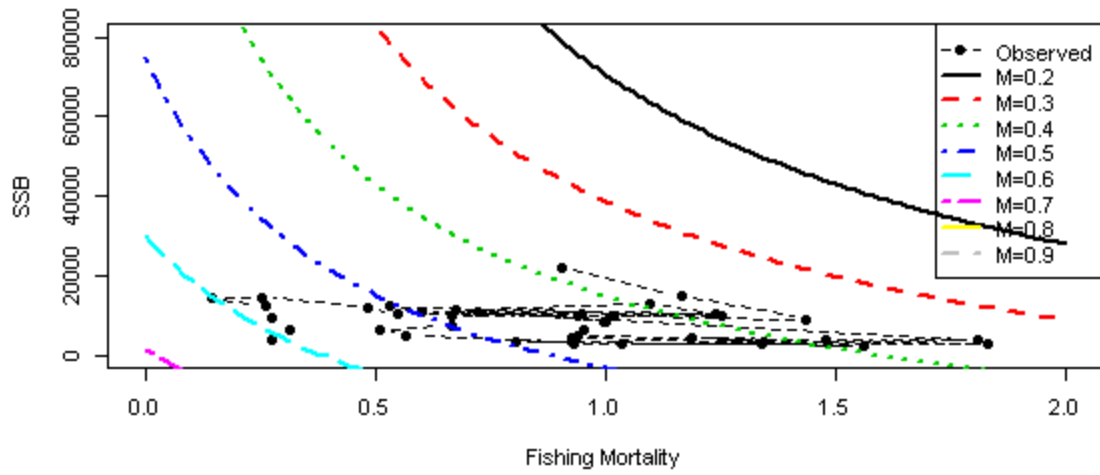


Figure 13. The ratio of  $B_{msy}$  to unfished SSB ( $B_0$ ) values associated with natural mortality rates ( $M$ ) and Beverton Holt stock recruitment curves for the Georges Bank yellowtail flounder example.

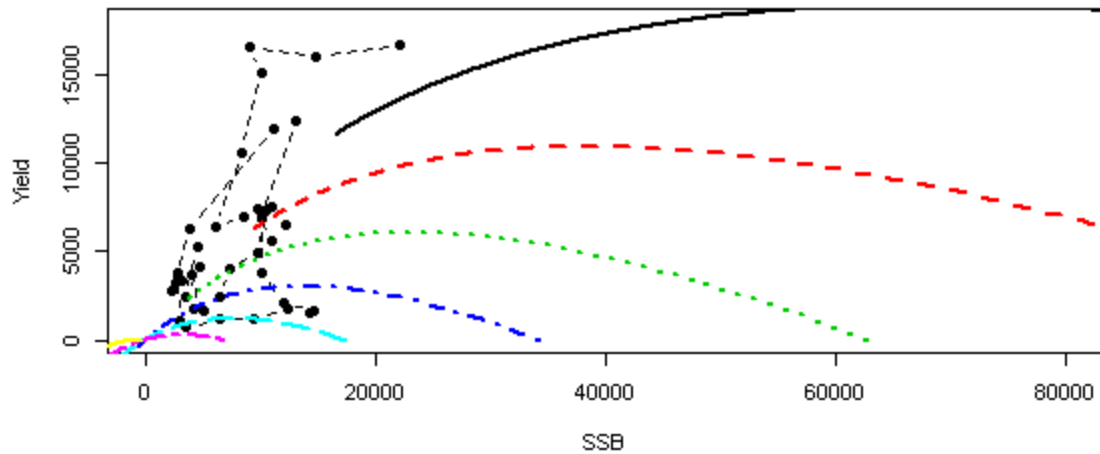
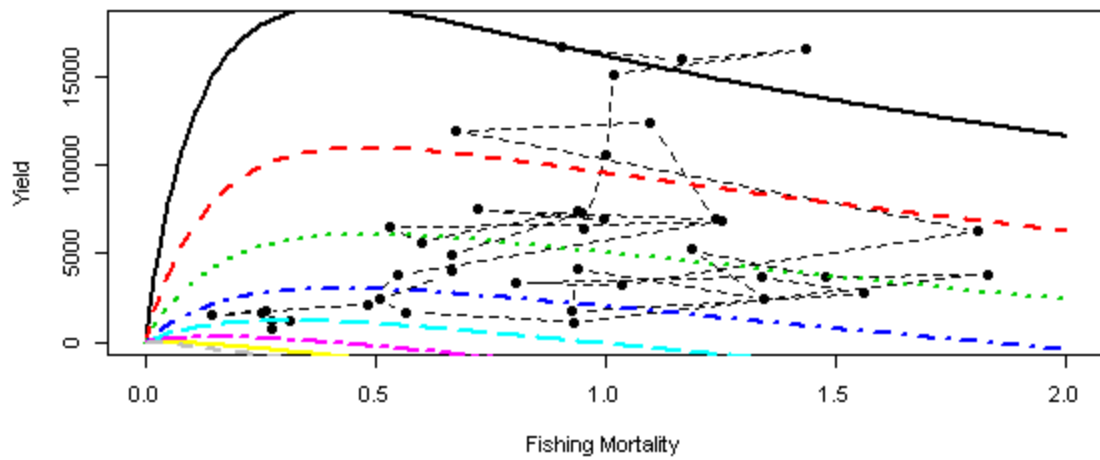
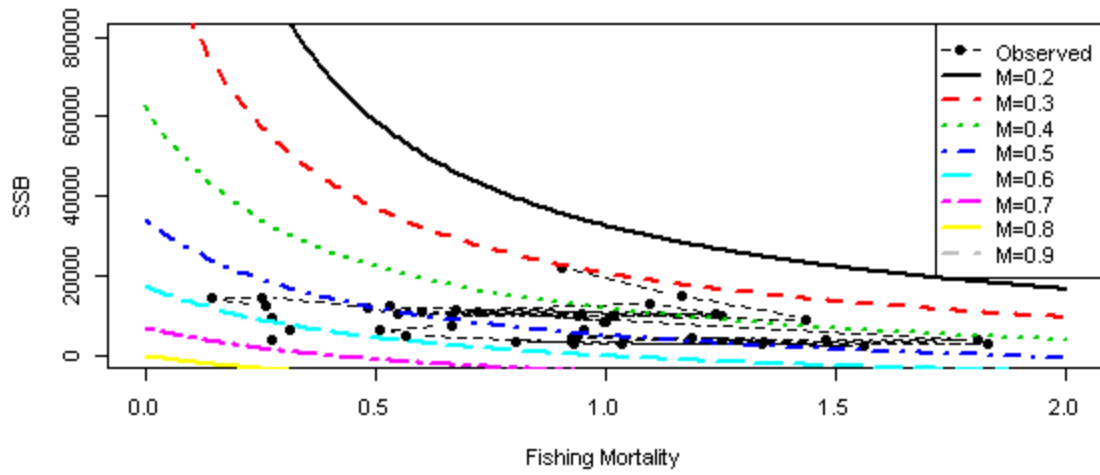
SR Curve 1



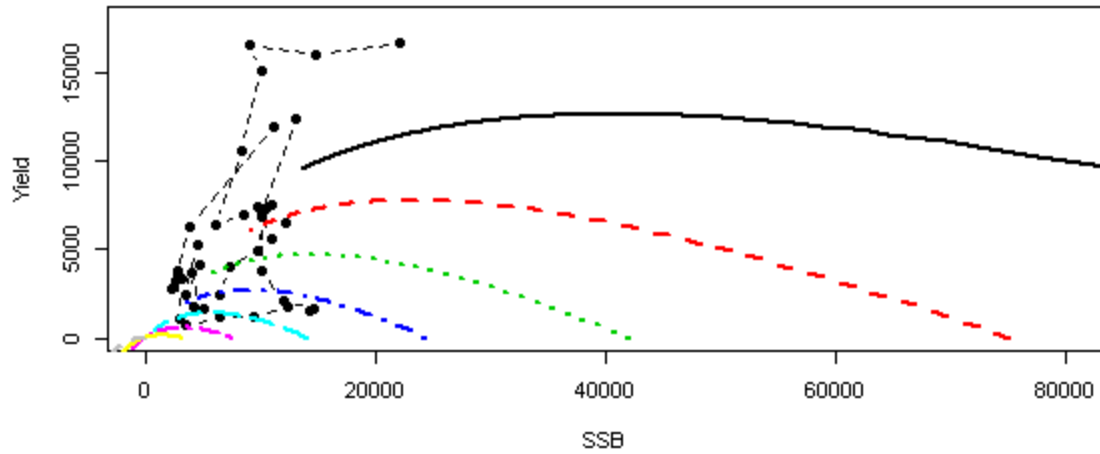
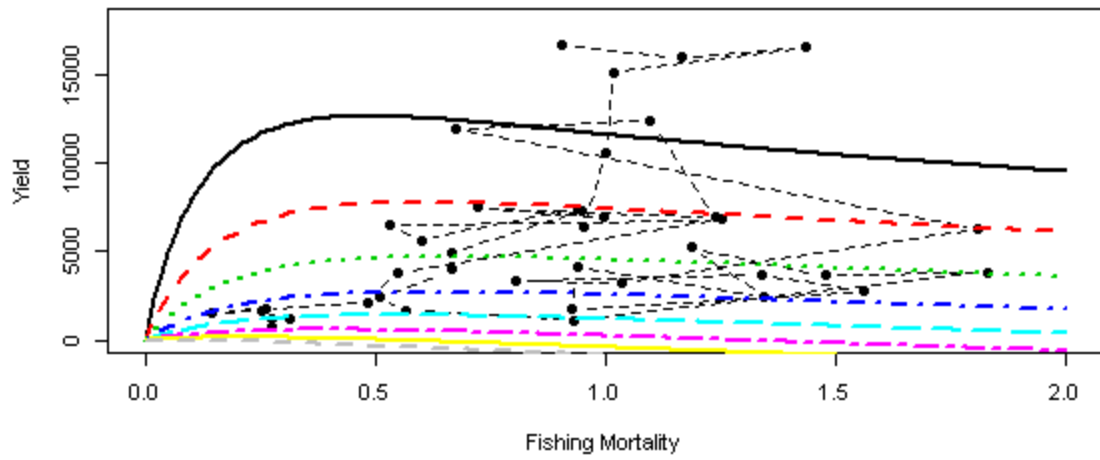
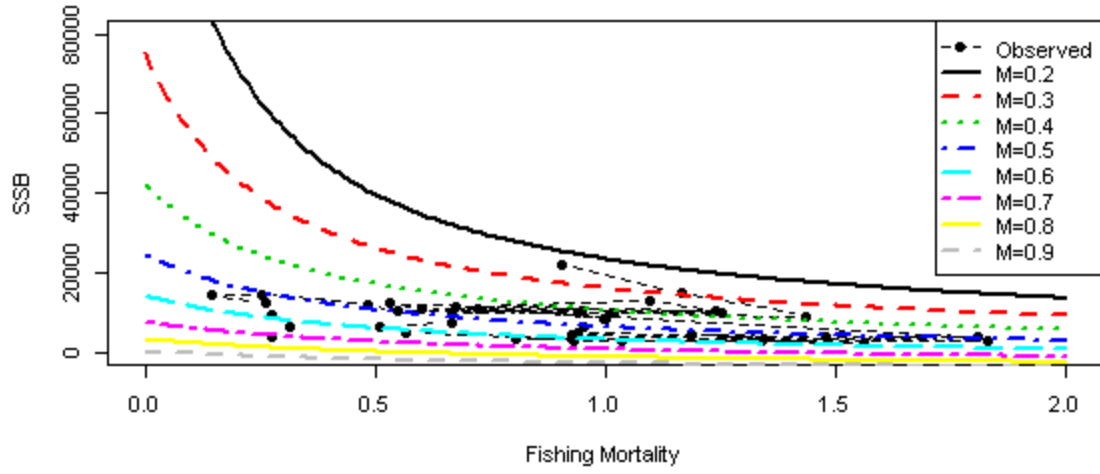
SR Curve 2



SR Curve 3

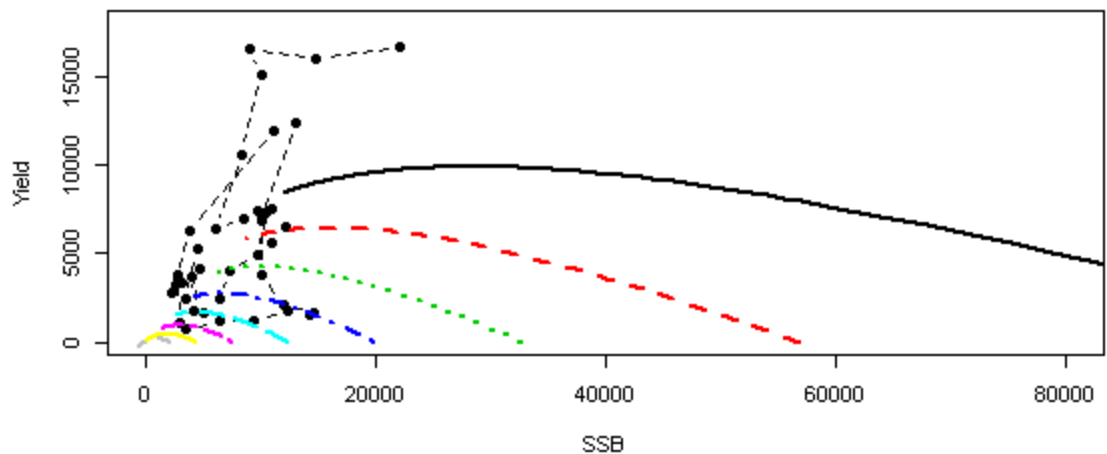
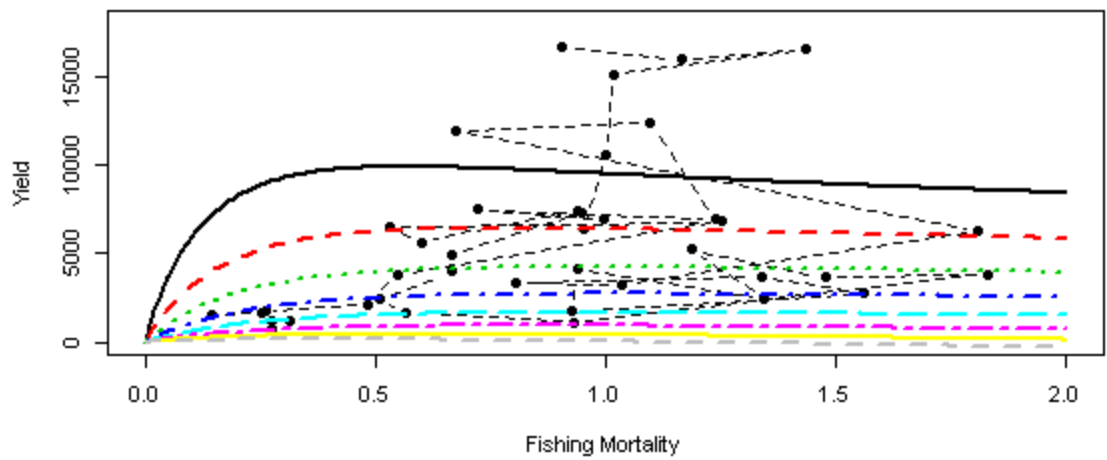
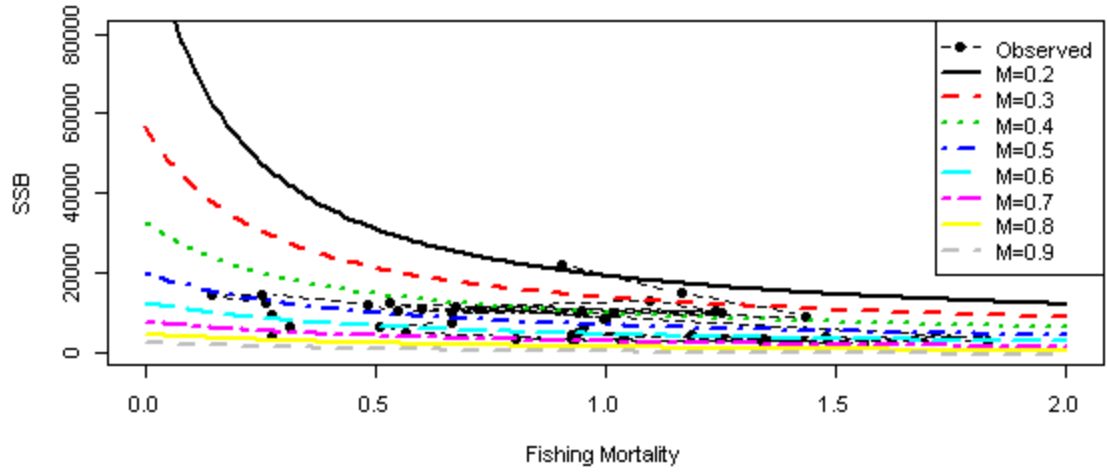


SR Curve 4





SR Curve 5



SR Curve 6

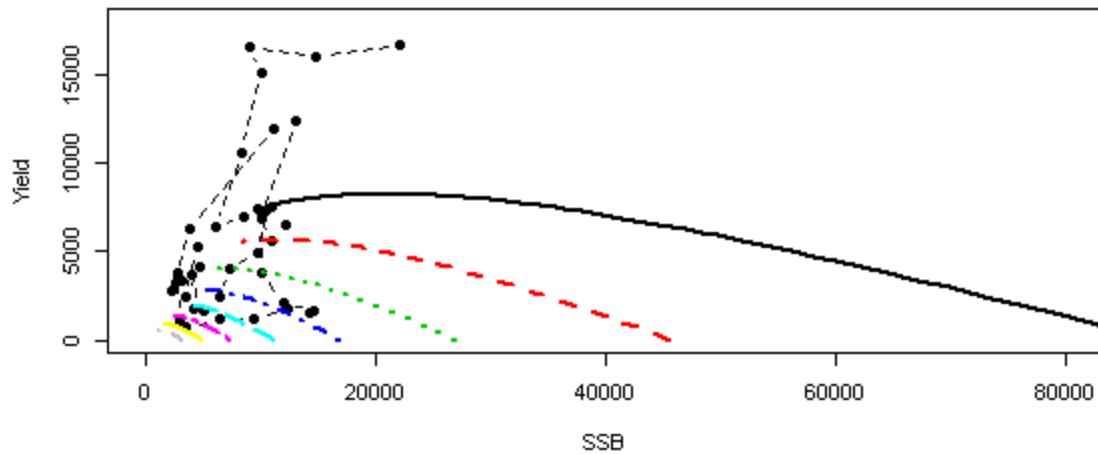
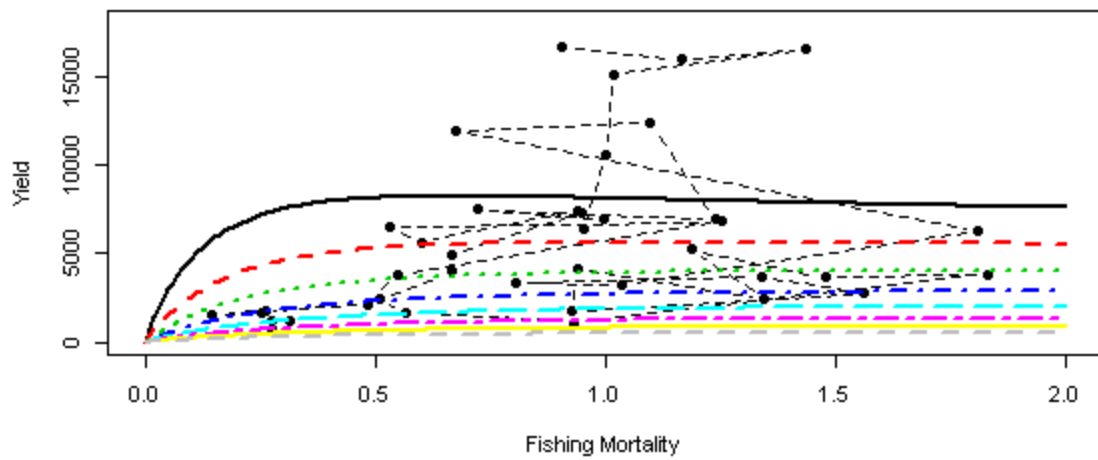
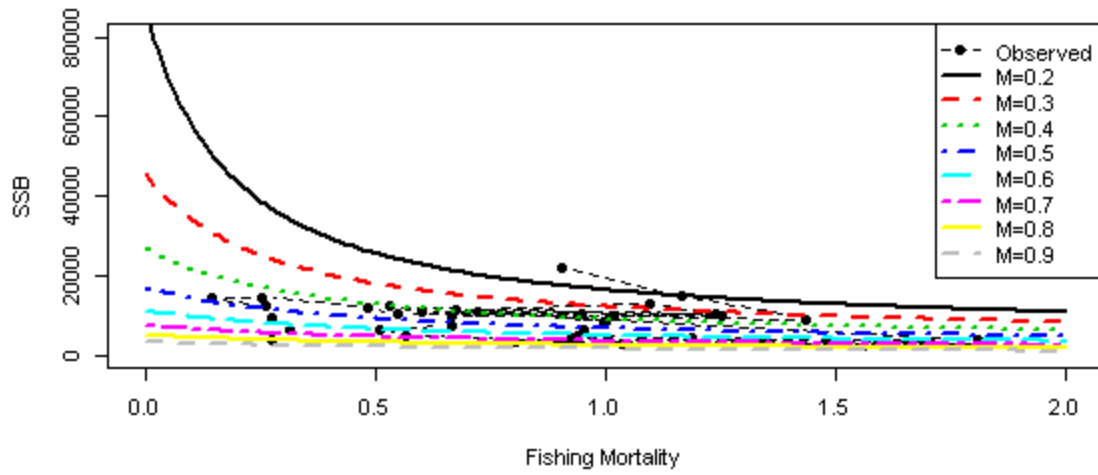


Figure 14. Spawning stock biomass versus fishing mortality rate (top panel), fishery yield versus fishing mortality rate (middle panel), and fishery yield versus spawning stock biomass (bottom panel) for the Georges Bank yellowtail flounder example. Each set of three panels corresponds to a different Beverton Holt SR curve. Within each panel, the black circles connected by dashed lines denote the VPA estimates (labeled observed) and the equilibrium values associated with a range of natural mortality values (colored lines with no symbols). Note the axes are consistent among the six sets to facilitate comparisons and the bottom panels do not show the equilibrium lines continuing to zero because only values of  $F$  up to 2.0 were used to make the plots. Values of  $F$  higher than 2 would be required to get the equilibrium lines in the bottom panel to be closer to the origin.